ISSN 1068-3712, Russian Electrical Engineering, 2011, Vol. 82, No. 1, pp. 38–43. © Allerton Press, Inc., 2011.
Original Russian Text © N.V. Donskoi, 2011, published in Elektrotekhnika, 2011, No. 1, pp. 40–46.

Three-Phase Mathematical Model of Induction Motor

N. V. Donskoi

Received December 16, 2010

Abstract—A three-phase mathematical model of induction motor based on description of processes in all three phases of the motor is considered. The factor that unites all three phases is the resulting magnetomotive force created by the stator and rotor currents and the main magnetic flux that passes through the air gap and rotates in the plane of the cross section of the induction motor.

Keywords: induction motor, mathematical model, cross-section plane, magnetomotive force, main magnetic flux.

DOI: 10.3103/S1068371211010032

Upon the creation of controllable AC electric drives, especially with high-voltage induction motors, the problem of reducing expenses for manufacturing and testing different variants of prototypes is quite topical. Computer simulation is of great help in solving this problem. In this case, an adequate mathematical model of induction motor that takes into account all of the most important properties of the motor plays an important role in the saturation of magnetic system, current displacement in rotor coils, and a number of other properties. So-called "virtual" models of induction motors can be created based on such a model; these models, in combination with models of conversion devices, make it possible to perform studies and tests with accuracy and results no worse than full-scale experiments.

The model should also make it possible to simulate the power supply of induction motor coils from different converters, including those with a nonsinusoudal output voltage and the power supply of phase coils from independent sources.

The best-known mathematical models are based on the description of processes in the two-dimensional state space using differential equations in vector form. In this case, all variables of the induction motor are represented by vectors that rotate in the cross-section plane of the motor. This plane is sometimes considered to be a complex plane, which makes it possible to reduce calculations by applying complex numbers. In this case, the sum of instantaneous values of phase voltages and currents must be equal to zero. The generalized AC machine or two-phase induction motor is studied for practical applications. In most cases, this approach is reasonable and yields good results.

The description of processes in three-dimensional (with orthogonal axes) state space is also known [1]. In this case, two axes are in the cross-section plane and the third axis coincides with the shaft axis. This is because the coordinates of the vector on the plane that show some variable, i.e., the projections of the vector onto the three axes of phases A, B, C electrically shifted by 120°, are linearly dependent. It is sufficient to determine two coordinates in order to unambiguously determine the position of the vector on the plane. In the general case, to simulate the induction motor, it is necessary for all three coordinates to be linearly independent.

This contradiction is overcome if a three-phase mathematical model is created that describes the processes in all three phases of an induction motor. The factor that unites all three phases is the resulting magnetomotive force created by the currents of stator and rotor phases and, correspondingly, the main magnetic flux that passes through the air gap.

This paper considers a mathematical model of an induction motor based on the description of processes in the cross-section plane of the motor.

Let arbitrary voltages $u_{1i}(t)$, where the phase index $i = A$, B, or C, be applied to the coils of stator phases of induction motor. These voltages create the currents $i_{1i}(t)$ in the stator phases. Based on Kirchoff's second law, it can be written that

$$u_{1i}(t) = r_1 i_{1i}(t) + \frac{d\psi_{1i}(t)}{dt}, \quad (1)$$

where r_1 is the active resistance, $\psi_{1i} = \psi_{1\sigma i} + \psi_{0i}$ is the total flux linkage of phases equal to the sum of scattering flux linkages $\psi_{1\sigma i} = l_{1\sigma} i_{1i}$ and air gap flux linkages ψ_{0i} connected with the main magnetic flux of induction motor Φ_0 created by combined action of magnetomotive force of the rotor and stator phases, and $l_{1\sigma}$ is the scattering inductance of stator phase coils.

A similar equation can be written for the rotor based on Park–Gorev equations as follows:

$$u_{2i}(t) = r_2 i_{2i}(t) + \frac{d\psi_{2i}(t)}{dt} - \omega_{\text{r.el}}\psi_{2i\text{ort}}(t), \quad (2)$$

where r_2 is active resistance, $i_{2i}(t)$ are the currents of the rotor phases, $\psi_{2i} = \psi_{2\sigma i} + \psi_{0i}$ is the total flux linkage of phases equal to the sum of scattering flux linkages $\psi_{2\sigma i} = l_{2\sigma} i_{2i}$ and flux linkages of the air gap ψ_{0i}, and $l_{2\sigma}$ is the scattering inductance of coils of the rotor phases.

In Eq. (2), all variables and parameters are reduced to the stator using known relations [2].

The term $\psi_{\text{r.el}}\psi_{2i\text{ort}}$ in (2) is the rotation electromotive force, which occurs as a result of reducing the known relations to the stator, which change with the angular frequency $\omega_2 = \omega_1 - \omega_{\text{r.el}}$, where ω_1 is the angular frequency of the stator, $\omega_{\text{r.el}}$ is the electric angular velocity of the rotor, $\psi_{2i\text{ort}}$ are the orthogonal flux linkages of the rotor, i.e., the flux linkages of the rotor at an angular distance from the axis of the rotor phase equal to $\pi/2$.

The sum of currents of the stator and rotor phases creates the resulting magnetomotive force $f_0(t, \vartheta)$, which is a function of time and the spatial angle on the electric plane of the motor cross section.

It is known that, at each point N on the air-gap circle, the values of the first harmonic of the magnetomotive-force phase, called the “spatial harmonic” are determined [2] as

$$f_{01i}(t, \vartheta) = \frac{2w_1 k_{\text{coil1}}}{\pi Z_p} i_{0i}(t)\cos(\vartheta_N - \vartheta_i), \quad (3)$$

where w_1 is the number of turns of the stator coil, $k_{\text{coil1}} \le 1$ is the coil coefficient that takes into account the reduction in the magnetomotive force in the case of the coil in the grooves, Z_p is the number of pole pairs, ϑ_N is the spatial angle corresponding to the observation point N, and ϑ_i is the spatial angle of the axis of ith phase on electric plane counted from the axis of phase A in the positive (counterclockwise) direction.

In this case, the angles ϑ_i are determined as $\vartheta_A = 0$, $\vartheta_B = 2\pi/3$, $\vartheta_C = 4\pi/3$. The currents of phase-magnetization circuits are $i_{0i}(t) = i_{1i}(t) + i_{2i}(t)$.

The spatial harmonic phases have the form of sinusoidal waves symmetric with respect to the phase axes. These waves do not change their spatial position; however, their amplitude changes in time following the phase currents and changes sign if the current polarities change.

The sum of spatial harmonic phases form a rotating spatial wave of the resulting magnetomotive force of the air gap as follows:

$$f_0(t, \vartheta) = f_{01A}(t, \vartheta) + f_{01B}(t, \vartheta) + f_{01C}(t, \vartheta).$$

Substituting (3) and the values of the angles ϑ_i, we obtain

$$f_0(t, \vartheta) = f_{0a}(t, \vartheta) + f_{0b}(t, \vartheta) = \frac{2w_1 k_{\text{coil1}}}{\pi Z_p} \times [i_{0ae}(t)\cos\vartheta_N + i_{0be}(t)\sin\vartheta_N], \quad (4)$$

where $i_{0ae}(t) = i_{0Ae}(t) - \frac{1}{2}[i_{0B}(t) + i_{0C}(t)]$, $i_{0be}(t) = \frac{\sqrt{3}}{2}[i_{0B}(t) + i_{0C}(t)]$ are the projections of the vector of equivalent current of the motor magnetization circuit onto orthogonal axes a and b corresponding to the real and imaginary axes of the complex plane; in this case, axis a coincides with axis A.

These projections are equivalent to the currents of two phases of equivalent two-phase induction motor. These projections are linearly independent and the resulting magnetomotive force of the air gap of the induction motor should be determined from these projections.

On the motor cross-section plane, the direction of the vector of equivalent current $I_{0e} = i_{0ae} + ji_{0be}$ coincides with the maximum resulting wave of the magnetomotive force and rotates with it at an angular velocity equal to the angular frequency of the stator currents.

The main magnetic flux Φ_0 induces the electromotive force in all stator and rotor coils. The electromotive force is also induced in any other conducting circuit in which magnetic flux changes. In a motor, these circuits include magnetic circuits of the stator and rotor. Since the plates of electric steel that make up the magnetic circuits have finite thickness and can be insufficiently isolated from each other, there are circuits for eddy currents that create energy losses.

Moreover, additional energy losses (i.e., hysteresis losses) that depend on the remagnetization frequency and magnetic induction occur due to the remagnetization of the stator and rotor plates. These two types of losses in steel are covered by part of the resulting magnetomotive force as follows:

$$f_0(t, \vartheta) = f_\mu(t, \vartheta) + f_{\text{Fe}}(t, \vartheta), \quad (5)$$

where f_μ is the reactive component of magnetomotive force that creates the magnetic flux Φ_0 and f_{Fe} is the active component which determines losses in steel.

The equivalent current of the magnetization circuit can be represented in a similar way as follows:

$$i_{0e}(t) = i_{\mu e}(t) + i_{\text{Fee}}(t).$$

The current defining the power of losses i_{Fee} in steel of two-phase induction motor is determined in terms of electromotive forces of phases induced by the main magnetic flux and the active resistance r_{Fee},

$$i_{\text{Fe}ae} = \frac{u_{0a}}{r_{\text{Fee}}} = \frac{d\psi_{0a}}{r_{\text{Fee}}dt} \quad \text{and } i_{\text{Fe}be} = \frac{u_{0b}}{r_{\text{Fee}}} = \frac{d\psi_{0b}}{r_{\text{Fee}}dt}.$$

For the same losses of the three-phase induction motor, we can write the following: $i_{\text{Fe}i} = \frac{u_{0i}}{r_{\text{Fe}}} = \frac{d\psi_{0i}}{r_{\text{Fe}}dt}$, where $r_{\text{Fe}} = 3/2r_{\text{Fee}}$. The resistance r_{Fe} with small error can be determined in terms of the power of losses in steel of the whole motor in the nominal regime P_{Fen} as follows:

$$r_{\text{Fe}} = \frac{3E^2_{0\text{ph.n}}}{P_{\text{Fen}}} \approx \frac{3U^2_{\text{ph.n}}}{P_{\text{Fen}}}. \tag{6}$$

Denoting (as in most sources) the mutual inductance of the stator and rotor coils by $l_m = \frac{\psi_{0i}}{i_{\mu i}}$, similar to r_{Fee}, we assume that $l_{me} = \frac{2}{3}l_m = \psi_{0i}/i_{\mu ie}$. Then, the equivalent current

$$i_{0i\text{e}} = \frac{3}{2l_m}\psi_{0i} + \frac{3}{2r_{\text{Fe}}}\frac{d\psi_{0i}}{dt}.$$

Then, substituting the symbol of differentiation p instead of d/dt, we obtain

$$\psi_{0i} = \frac{2l_m}{3}\frac{1}{1+T_\mu p}i_{0i\text{e}} = \frac{2l_m}{3}i_{\mu i\text{e}},$$

where $T_\mu = \frac{l_m}{r_{\text{Fe}}} = l_{me}/r_{\text{Fee}}$ is the time constant of the magnetization circuit, $i_{\mu ie} = \frac{1}{T_\mu p + 1}i_{0ie}$.

Using the Laplace transform, we find that the flux linkage of the phase air gap is connected with the equivalent current of the magnetization circuit by the transfer function of the inertial section, i.e.,

$$\psi_{0i}(p) = \frac{2l_m}{3}\frac{I_{0i\text{e}}(p)}{1+T_\mu p}. \tag{7}$$

If the stator coils in the motor are connected into a star and there is no connection between the common point of the coils and the common point of the power-supply source, the equivalent currents are equal to 3/2 of phase currents, and formula (7) takes the simpler form

$$\psi_{0i}(p) = \frac{l_m I_{0i}(p)}{1+T_\mu p}. \tag{8}$$

Formulas (7) and (8) yield the important conclusion that for determination of flux linkage of the air gap it is not necessary to find the reactive component of current of the magnetization circuit. Upon simulation of the induction motor it is sufficient to make the current of this circuit flow through the inertial section with the time constant T_μ.

The wave of reactive component of the magnetomotive force determines the spatial distribution of magnetic induction in the air gap. This component can be determined as follows based on the law of the total current and neglecting magnetic field strength:

$$b_\delta(t, \vartheta) = \frac{f_\mu(t,\vartheta)\mu_0}{\delta} = \frac{2w_1k_{\text{coil}1}\mu_0}{\pi Z_p\delta} \times [i_{\mu a\text{e}}(t)\cos\vartheta_N + i_{\mu b\text{e}}(t)\sin\vartheta_N],$$

where δ is the radial length of the air gap.

To find the phase flux linkage, it is necessary to find the average value of magnetic induction on the pole pitch under the phase coils. Mathematical transformations yield the following expressions of average induction for the given time instant:

$$B_{\delta A\text{av}} = \frac{4w_1k_{\text{coil}1}\mu_0}{\pi^2\delta Z_p}i_{\mu a\text{e}};$$

$$B_{\delta B\text{av}} = \frac{4w_1k_{\text{coil}1}\mu_0}{\pi^2\delta Z_p}\left[-\frac{1}{2}i_{\mu a\text{e}}(t) + \frac{\sqrt{3}}{2}i_{\mu b\text{e}}(t)\right];$$

$$B_{\delta C\text{av}} = \frac{4w_1k_{\text{coil}1}\mu_0}{\pi^2\delta Z_p}\left[-\frac{1}{2}i_{\mu a\text{e}}(t) - \frac{\sqrt{3}}{2}i_{\mu b\text{e}}(t)\right].$$

In this case, the phase flux linkage $\psi_{0i} = w_1k_{\text{coil}1}S_\tau B_{\delta i\text{av}}$, where $S_\tau = \tau l$ is the pole area and l is the active length of the stator.

Assuming, according to [2], that $l_m = \frac{\psi_{0i}}{i_{\mu i}} = \frac{6w_1^2k^2_{\text{coil}1}\mu_0S_\tau}{\pi^2Z_p\delta}$, we obtain

$$\psi_{0A} = \frac{2}{3}l_mi_{\mu A\text{e}};\quad \psi_{0B} = \frac{2}{3}l_mi_{\mu B\text{e}};\quad \psi_{0C} = \frac{2}{3}l_mi_{\mu C\text{e}}, \tag{9}$$

where $i_{\mu A\text{e}} = i_{\mu a\text{e}} = i_{1\mu A} - \frac{1}{2}(i_{1\mu B} + i_{1\mu C})$;

$$i_{\mu B\text{e}} = i_{1\mu B} - \frac{1}{2}(i_{1\mu A} + i_{1\mu C});$$

$$i_{\mu C\text{e}} = i_{1\mu C} - \frac{1}{2}(i_{1\mu B} + i_{1\mu A}).$$

For the sum of phase currents equal to zero, we obtain the known formula $\psi_{0i} = l_mi_{\mu i}$.

The magnetic system saturation can be taken into account by the averaged characteristic of the motor magnetization. Let us denote the amplitude of the first harmonic of phase flux linkage of the air gap with account of saturation by $\psi_{0m\text{sat}}$, and the amplitude of flux linkage in the absence of saturation for the same magnetization current by ψ_{0m}. Based on data for SC motors [3], we construct the average dependences of relative flux linkage $\psi_{0m\text{sat}}$ and the saturation coefficient $K_{\text{sat}} = \frac{\psi_{0m\text{sat}}}{\psi_{0m}}$ on the relative magnetization current $I_{\mu im*} = I_{\mu im}/I_{\mu im\text{nom}}$ (Fig. 1). In Fig. 1, in the nominal regime for $I_{\mu im*} = 1$ the relative saturation flux linkage $\psi_{0m\text{sat}*} = \psi_{0m\text{sat}}/\psi_{0m\text{nom}}$ is also equal to unity.

Since $\psi_{0m*} = \psi_{0m}/\psi_{0m\text{nom}} = l_m I_{\mu im}/l_m I_{\mu im\text{nom}}$, the dependence $K_{\text{sat}} = f(T_{\mu im*})$ is also the dependence $K_{\text{sat}} = f(\psi_{0m*})$. Using this dependence for the known ψ_{0m*}, we find K_{sat} and $\psi_{0i\text{sat}} = \psi_{0i}K_{\text{sat}}$.

The total flux linkages of the stator (with account of scattering flux linkages) for the phases are found by integrating (1), $\psi_{1i} = \int_0^t (u_{1i} - r_1 i_{1i})\,dt$.

The current of the stator phases is determined by the scattering flux linkage $\psi_{1\sigma i} = \psi_{1i} - \psi_{0i\text{sat}}$ as $i_{1i} = \frac{\psi_{1\sigma i}}{l_{1\sigma}}$. The flux linkage of the rotor is determined similarly using (2).

Based on this, for the squirrel-cage induction motor and $u_{2i} = 0$, we write

$$\psi_{2i} = \int_0^t (-r_2 i_{2i} - \omega_{\text{r.el}}\psi_{2i\text{ort}})\,dt. \quad (10)$$

The orthogonal phase flux linkages are determined according to the following formulas [1]:

$$\psi_{2A\text{ort}} = \frac{\psi_{2B} - \psi_{2C}}{\sqrt{3}}; \quad \psi_{2B\text{ort}} = \frac{\psi_{2C} - \psi_{2A}}{\sqrt{3}}; \quad \psi_{2C\text{ort}} = \frac{\psi_{2A} - \psi_{2B}}{\sqrt{3}}. \quad (11)$$

Equations (7)–(11) form the basis for the mathematical model of the squirrel-cage induction motor. However, in this model, it is necessary to take into account the current displacement in the rotor coils. For this purpose, we consider two parameters r_2 and $l_{2\sigma}$ as separate variables.

In reference materials on motors [4], active and inductive rotor resistances are usually given at rated slip (we denote them by r_{20} and $x_{2\sigma 0} = l_{2\sigma 0}\omega_{\text{s.n}}$), as well as at slip equal to 1 (we denote them by r_{2k} and $x_{2\sigma k} = l_{2\sigma k}\omega_{\text{s.n}}$) and $r_{2k} > r_{20}$, $x_{2\sigma k} < x_{2\sigma 0}$. Here, $\omega_{\text{s.n}}$ is the synchronous nominal angular velocity of the rotor.

If the slip changes from 1 to 0, e.g., in the case of the direct connection of the motor to the power-supply network, the active resistance decreases and depends in a complex way on the slip, and the inductive resistance increases. In this case, the shape and depth of the rotor slot, the size of cross sections of the rotor coil conductors, etc. play an important role. In some references, e.g., [3], it is indicated that, with increasing slip s or, more precisely, the rotor current frequency or absolute slip

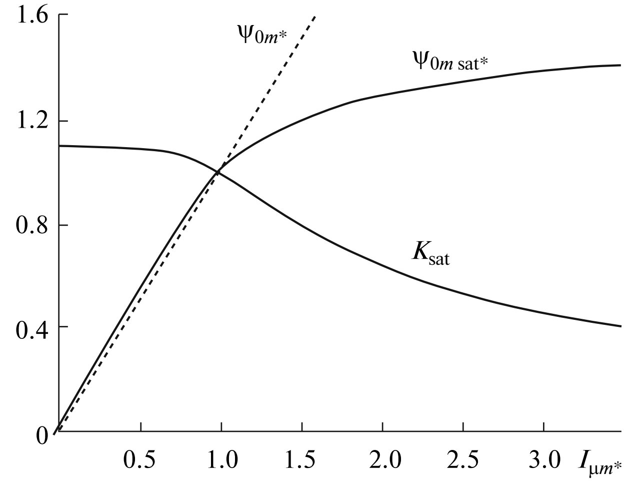

Fig. 1. $\psi_{0m\text{sat}*}$, K_{sat} as functions of $I_{\mu m*}$.

$\beta = (\omega_1 - \omega_2)/\omega_{\text{s.n}}$, the resistance r_2 increases in proportion to the square root of β. However, the comparison with the reference and experimental data shows that this is not always valid. In some cases, an inverse (quadratic) dependence is even observed.

In the general case, the change in the active resistance of the rotor can be approximately represented by the following dependence:

$$r_2 = r_{20} + (r_{2k} - r_{20})|\beta|^{k_r}, \quad (12)$$

where the exponent k_r can change from 0.5 to 2, or even 3.

Similarly, the inductive resistance is

$$x_{2\sigma} = x_{2\sigma 0} - (x_{2\sigma 0} - x_{2\sigma k})|\beta|^{k_x}, \quad (13)$$

where k_x can be equal to k_r or differ from it; in each particular case, these coefficients should be chosen according to the mechanical characteristic $M = f(\beta)$ for the established operating regime given in the reference books.

The motor torque can be found using the method [1] based on Ampere's law for a force acting on the conductor with current in a magnetic field and by applying orthogonal flux linkages. This method yields the following formula for the induction motor torque as follows:

$$M = Z_p[\psi_{0A\text{ort}}(t)i_{1A}(t) + \psi_{0B\text{ort}}(t)i_{1B}(t) + \psi_{0C\text{ort}}(t)i_{1C}(t)], \quad (14)$$

where

$$\psi_{0A\text{ort}} = \frac{\psi_{0C}(t) - \psi_{0B}(t)}{\sqrt{3}}; \quad \psi_{0B\text{ort}} = \frac{\psi_{0A}(t) - \psi_{0C}(t)}{\sqrt{3}};$$

$$\psi_{0C\text{ort}} = \frac{\psi_{0B}(t) - \psi_{0A}(t)}{\sqrt{3}}$$

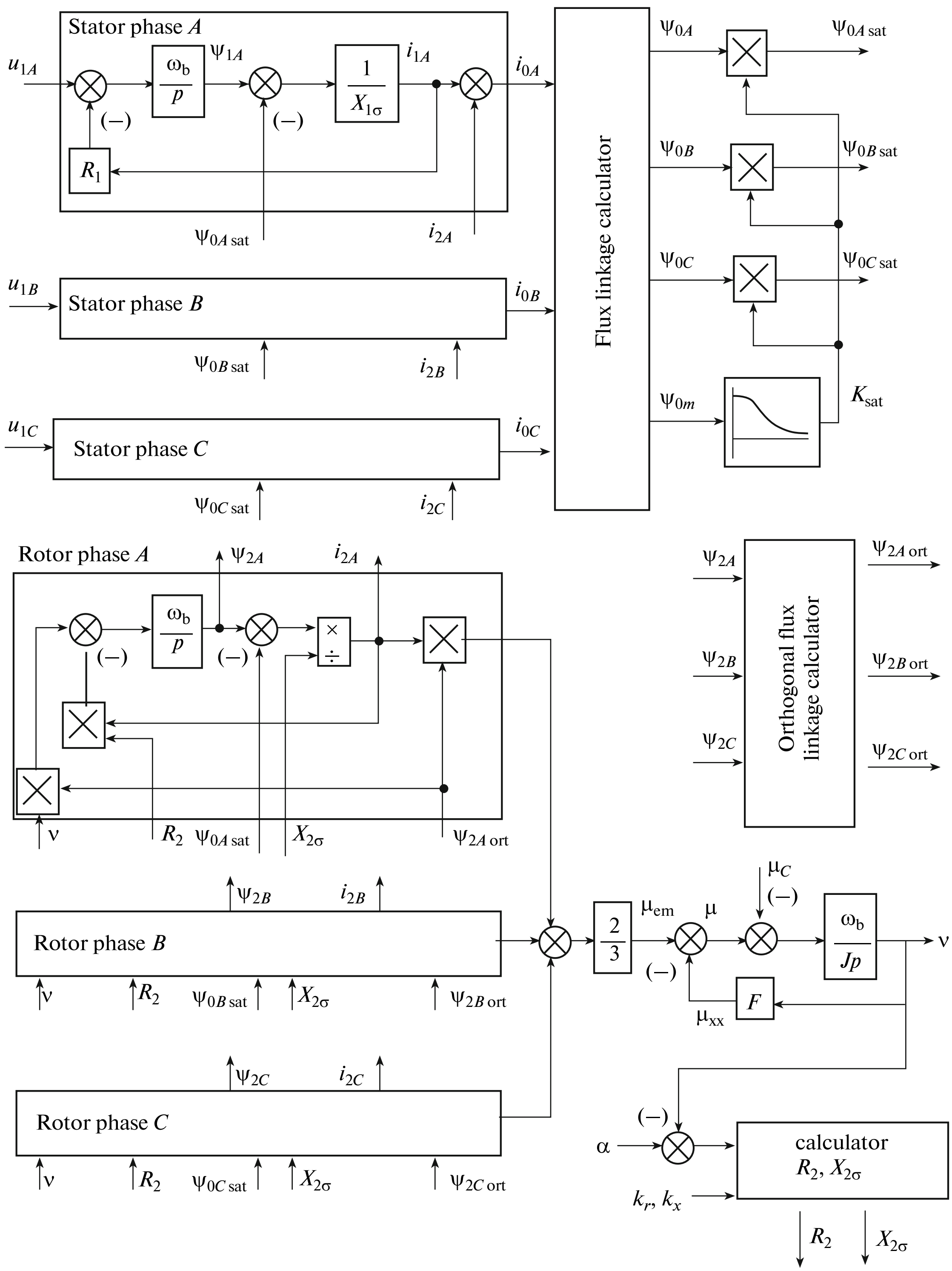

Fig. 2. Schematic diagram of mathematical model of squirrel-cage induction motor.

are the orthogonal flux linkages of the air gap.

Formula (14) demonstrates that the electromagnetic torque of the induction motor is determined by the sum of the products of phase currents and orthogonal flux linkages of the air gap. This is a universal formula. In this formula, the phase currents of the stator can be replaced by rotor currents and rotor flux linkages can be used with the sign of orthogonal flux link-

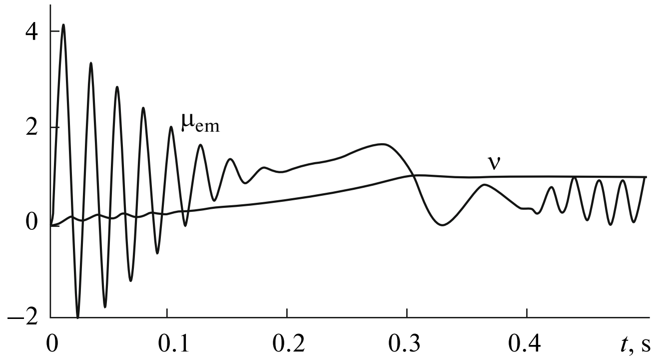

Fig. 3. Direct actuation of 4A180M6U3-type induction motor.

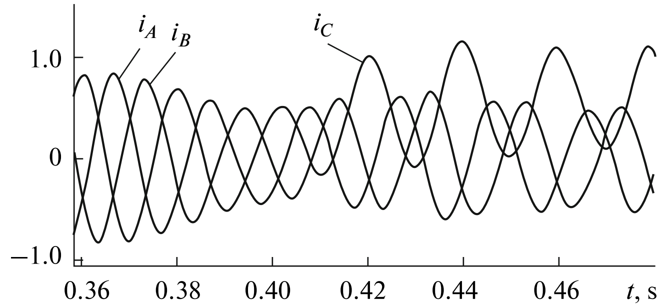

Fig. 4. Phase currents of induction motor for direct actuation.

ages changed to the opposite one. This is preferable to use the rotor flux linkages, especially taking into account that they are already used in the determination of the rotation electromotive force. The result is the same.

The three-phase mathematical model of the induction motor can be made based on the formulas presented above. However, it is better to write this model in terms of relative units [1, 2]. We take the following as the basic quantities: the amplitudes of nominal phase voltages and currents, synchronous angular frequency $\omega_b = \omega_{s.n}$, basic torque $M_b = P_b Z_b/\omega_b$, and basic resistance $R_b = X_b = Z_b = U_b/I_b$. Below, we denotethe relative parameters with capital letters.

In relative units, the inductance and inductive resistance have the same value of $L = l/L_b = \omega_{s.n} l/\omega_{s.n} L_b = x/X_b = X$.

For the mathematical model (Fig. 2) to operate in real-time scale, it is necessary to put the amplification section with the coefficient ω_b in front of all integrators. The time constants of inertial sections should be reduced by the same factor.

In relative units, the number of pole pairs in the formula for the torque vanishes, but a coefficient of 2/3 appears, i.e., $\mu_{em} = M/M_b = (i_{2A}\psi_{2A\text{ort}} + i_{2B}\psi_{2B\text{ort}} + i_{2C}\psi_{2C\text{ort}}) \times 2/3$.

The flux linkages of the air gap for $L_m\omega_{s.n} = X_m$ are calculated using the formulas

$$\psi_{0A} = \frac{2}{3}X_m\left(i_{0A} - \frac{i_{0B} + i_{0C}}{2}\right)\frac{1}{T_\mu p + 1},$$

$$\psi_{0B} = \frac{2}{3}X_m\left(i_{0B} - \frac{i_{0A} + i_{0C}}{2}\right)\frac{1}{T_\mu p + 1},$$

$$\psi_{0C} = \frac{2}{3}X_m\left(i_{0C} - \frac{i_{0A} + i_{0B}}{2}\right)\frac{1}{T_\mu p + 1},$$

$$\psi_{0m} = \sqrt{\frac{2}{3}(\psi_{0A}^2 + \psi_{0B}^2 + \psi_{0C}^2)}.$$

Orthogonal flux linkages of the rotor are calculated using (11).

In the scheme in Fig. 2, $a = \omega_1/\omega_b$ is the relative angular frequency of the stator variables and $\nu = \omega_{r.el}/\omega_b$ is the relative angular velocity of the rotor.

To demonstrate the model performance, Figs. 3 and 4 show the oscillograms obtained in the simulation of the direct actuation of an induction motor of the 4A180M4U3 type for the static load on the shaft $0.5M_l$. At a time instant of 0.4 s, a direct voltage equal to $0.02U_{ph.n}$ was input into phase C of the motor. As expected, this resulted in the constant component in phase C current equal to the constant component of the voltage divided by the active resistance of the phase. Correspondingly, torque pulsations with a frequency of 50 Hz appeared. In other phases, the current was unchanged. In this case, the sum of currents did not vanish.

The data of oscillograms show the efficiency and adequacy of the presented model of the induction motor. Thus, the developed three-phase mathematical model of an induction motor makes it possible to study different operating regimes, including in the case of a stator coil powered from independent power-supply sources.

REFERENCES

1. Shreiner, R.T., *Matematicheskoe modelirovanie elektroprivodov peremennogo toka s poluprovodnikovymi preobrazovatelyami chastoty* (Mathematical Simulation of AC Drives with Semiconduction Frequency Converters), Yekaterinburg: URO RAN, 2000.
2. *Elektrotekhnicheskii spravochnik* (Handbook. Electricity), vol. 2: *Elektrotekhnicheskie izdeliya i ustroistva* (Electrotechnical Units and Devices), Gerasimov, V.G., Ed., Moscow: Energoatomizdat, 1986.
3. Kostenko, M.V. and Piotrovskii, L.M., *Elektricheskie mashiny* (Electrical Machines), part 2: *Mashiny peremennogo toka* (AC Electrical Machines), Leningrad: Energiya, 1973.
4. *Asinkhronnye dvigateli serii 4A: Spravochnik* (Induction Motors of 4A Series: Handbook), Kravchik, A.E., Shlaf, M.M., Afonin, V.I., and Sobolenskaya, E.A., Eds., Moscow: Energoizdat, 1982.